

Discrete Prob [79 marks]

1. [Maximum mark: 6]

The following table shows the probability distribution of a discrete random variable X where $x = 1, 2, 3, 4$.

x	1	2	3	4
$P(X=x)$	k^2	$7k+2$	$-2k$	$3k^2$

Find the value of k , justifying your answer.

[6]

2. [Maximum mark: 6]

The discrete random variable X has the following probability distribution:

x	1	2	3	4
$P(X=x)$	0.2	0.3	k	0.1

(a) Find the value of k .

[2]

(b) Find $P(X > 1.5)$.

[2]

(c) Find $E(X)$.

[2]

3. [Maximum mark: 13]

A discrete random variable, X , has the following probability distribution, where $a > 0$ and k is a constant.

x	0	a	$2a$	$3a$
$P(X=x)$	k	$3k^2$	$2k^2$	k^2

- (a) Show that $k = \frac{1}{3}$. [5]
- (b) Find $P(X < 3a)$. [2]
- (c) Find $P(X \geq a \mid X < 3a)$. [3]
- (d) Given that $E(X) = 20$, find the value of a . [3]

4. [Maximum mark: 6]

A supermarket analyses the shopping habits of its customers.

The number of times, X , each customer visits the supermarket in a week is given by the following probability distribution.

x	1	2	3	4	5	≥ 6
$P(X=x)$	$1.5a$	$2a$	0.281	a	0.026	0

- (a.i) Find the value of a . [2]
- (a.ii) Write down the mode of X . [1]
- (b) Find the mean of X . [2]

The manager wants to know why customers come to their supermarket. They survey the first 50 customers to arrive at the supermarket on a particular day.

- (c) Identify which one of the following best describes the manager's sampling method.

Circle your answer.

[1]

Simple random / Systematic / Convenience / Quota / Stratified

5. [Maximum mark: 5]

A discrete random variable, X , has the following probability distribution:

$$P(X = x) = \frac{kx}{15} \text{ for } x \in \{2, 4, 7, 10\}.$$

(a) Find the value of k . [2]

(b) Find $E(X)$. [3]

6. [Maximum mark: 5]

A discrete random variable, X , has the following probability distribution:

$$P(X = x) = \frac{kx}{20} \text{ for } x \in \{3, 5, 8, 11\}.$$

(a) Find the value of k . [2]

(b) Find $E(X)$. [3]

7. [Maximum mark: 17]

A bag contains buttons which are either red or blue.

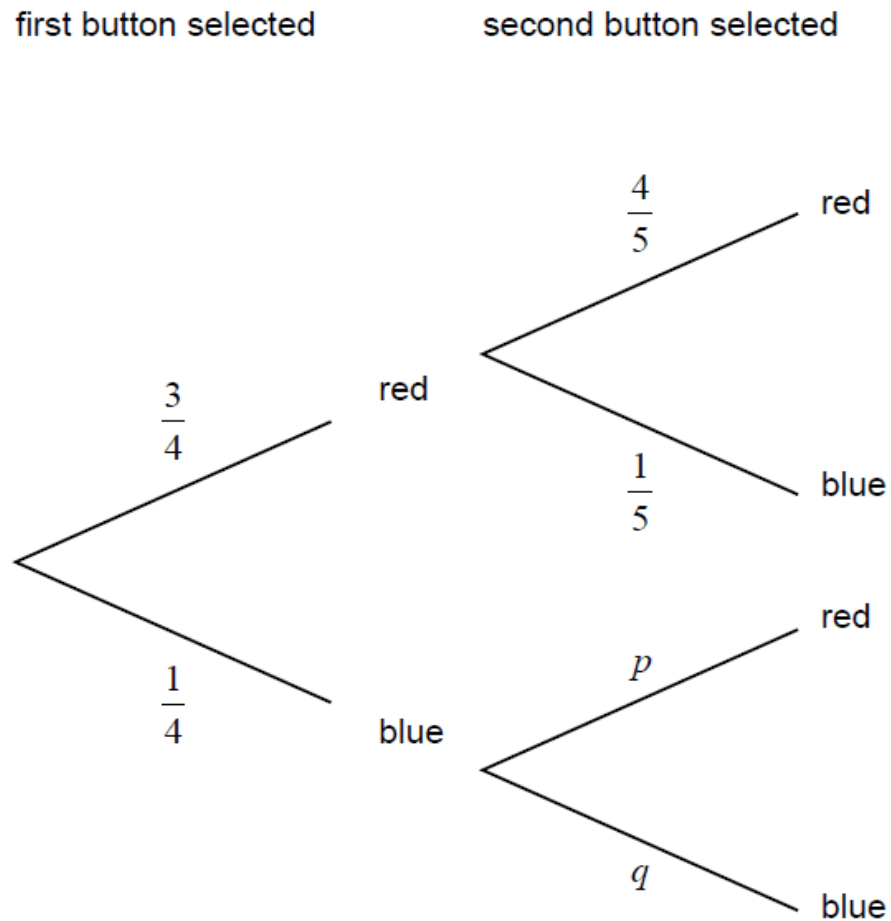
Initially, the bag contains three red buttons and one blue button.

Francine randomly selects one button from the bag. She then replaces the button and adds one extra button of the same colour.

For example, if she selects a red button, she then replaces it and adds one extra red button so that the bag then contains four red buttons and one blue button.

Francine then randomly selects a second button from the bag.

The following tree diagram represents the probabilities of the first two selections.



- (a) Find the value of p and the value of q . [2]
- (b) Show that the probability that Francine selects two buttons of the same colour is $\frac{7}{10}$. [2]
- (c) Given that Francine selects two buttons of the same colour, find the probability that she selects two red buttons. [3]

The random variable X is defined as the number of red buttons selected by Francine.

The following table shows the probability distribution of X .

x	0	1	2
$P(X = x)$	$\frac{1}{10}$	a	b

(d) Find the value of a and the value of b . [2]

(e) Hence, find the expected number of red buttons selected by Francine. [2]

Francine restarts the process with three red buttons and one blue button in the bag. She selects buttons as before, replacing the button and adding one extra button of the same colour each time. She repeats this until she selects a blue button.

(f) Given that the first two buttons she selects are red, write down the probability that the next button she selects is blue. [1]

The probability that she selects the first blue button after n selections in total is $\frac{3}{56}$.

(g) Find the value of n . [5]

8. [Maximum mark: 4]

Events A and B are such that $P(A) = 0.7$, $P(B) = 0.75$ and $P(A \cap B) = 0.55$.

(a) Find $P(A \cup B)$. [2]

(b) Hence, otherwise find $P(A \cap B)$. [2]

9. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X , where $a, k \in \mathbb{R}^+$.

x	1	2	3	4
$P(X=x)$	k	k^2	a	k^3

Given that $E(X) = 2.6$, find the value of a . [5]

10. [Maximum mark: 6]

Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

(a) Find the common difference, d . [2]

The following table shows the probability distribution of a discrete random variable X such that $P(X = n) = \frac{u_n}{k}$, where $n \in \mathbb{Z}^+, 1 \leq n \leq 4$ and $k \in \mathbb{R}^+$.

n	1	2	3	4
$P(X = n)$	$\frac{0.6}{k}$	$\frac{u_2}{k}$	$\frac{u_3}{k}$	$\frac{0.15}{k}$

(b) Find the value of k . [4]

11. [Maximum mark: 6]

A discrete random variable, X , has the following probability distribution:

x	0	1	2	3
$P(X=x)$	0.41	$k - 0.28$	0.46	$0.29 - 2k^2$

- (a) Show that $2k^2 - k + 0.12 = 0$. [1]
- (b) Find the value of k , giving a reason for your answer. [3]
- (c) Hence, find $E(X)$. [2]